Compound Interest

Finite Math

15 February 2017

Quiz

If P dollars is invested in a savings account with an annual simple interest rate r, how much is in the account after t years?

$$A = ???$$

Average Daily Balance

Recall that the Average Daily Balance method is done by

Average Daily Balance

Recall that the Average Daily Balance method is done by

1. Finding the balance at the end of each day

Average Daily Balance

Recall that the Average Daily Balance method is done by

1. Finding the balance at the end of each day
2. Adding up the daily balances

Average Daily Balance

Recall that the Average Daily Balance method is done by

1. Finding the balance at the end of each day
2. Adding up the daily balances
3. Dividing by the total number of days (the ADB)

Average Daily Balance

Recall that the Average Daily Balance method is done by

1. Finding the balance at the end of each day
2. Adding up the daily balances
3. Dividing by the total number of days (the ADB)
4. Compute the interest based on the ADB

Average Daily Balance

Recall that the Average Daily Balance method is done by

1. Finding the balance at the end of each day
2. Adding up the daily balances
3. Dividing by the total number of days (the ADB)
4. Compute the interest based on the ADB
5. Add the interest to the final day's balance as found in (1).

Now You Try It!

Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

Now You Try It!

Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

Solution

\$708.92

Compound Interest

In the case of simple interest, the interest is computed exactly once:

Compound Interest

In the case of simple interest, the interest is computed exactly once: at the end.

Compound Interest

In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Compound Interest

In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example

Suppose $\$5,000$ *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

Compound Interest

If we generalize this process, we end up with the following result

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \ \textit{where } i = \frac{r}{m}$$

The variables in this equation are

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- A = *future value after n compounding periods*

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- A = *future value after n compounding periods*
- P = *principal*

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- $A =$ *future value after n compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1 + i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- A = *future value after n compounding periods*
- P = *principal*
- r = *annual nominal rate*
- m = *number of compounding periods per year*

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- $A =$ *future value after n compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $i =$ *rate per compounding period*

Compound Interest

If we generalize this process, we end up with the following result

Definition (Compound Interest)

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- A = *future value after n compounding periods*
- P = *principal*
- r = *annual nominal rate*
- m = *number of compounding periods per year*
- i = *rate per compounding period*
- n = *total number of compounding periods*

Compound Interest

Alternately, one can reinterpret this formula as a function of time as

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A, P, r, and m have the same meanings as above and t is the time in years.

Compound Interest

Example

If $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Now You Try It!

Example

If $\$2,000$ is invested at 7% compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after 5 years? How much interest is accrued in each case? Round answers to the nearest cent.

Now You Try It!

Example

If $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after* 5 *years? How much interest is accrued in each case? Round answers to the nearest cent.*

Solution

(a) $\$2805.10$ *with* $\$805.10$ *in interest.*

(b) $\$2829.56$ *with* $\$829.56$ *in interest.*

(c) $\$2835.25$ *with* $\$835.25$ *in interest.*

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned} A &= P\left(1 + \frac{r}{m}\right)^{mt} \\ &= P\left(1 + \frac{r}{m}\right)^{mt \cdot \frac{r}{r}} \end{aligned}$$

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned} A &= P\left(1 + \frac{r}{m}\right)^{mt} \\ &= P\left(1 + \frac{r}{m}\right)^{mt \cdot \frac{r}{r}} \\ &= P\left(1 + \frac{r}{m}\right)^{\left(\frac{m}{r}\right)rt} \end{aligned}$$

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned} A &= P\left(1 + \frac{r}{m}\right)^{mt} \\ &= P\left(1 + \frac{r}{m}\right)^{mt \cdot \frac{r}{r}} \\ &= P\left(1 + \frac{r}{m}\right)^{\left(\frac{m}{r}\right)rt} \\ &= P\left(1 + \frac{1}{x}\right)^{xrt} \qquad \left(x = \frac{m}{r}\right) \end{aligned}$$

Continuous Compound Interest

Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned}
A &= P\left(1 + \frac{r}{m}\right)^{mt} \\
&= P\left(1 + \frac{r}{m}\right)^{mt\cdot\frac{r}{r}} \\
&= P\left(1 + \frac{r}{m}\right)^{\left(\frac{m}{r}\right)rt} \\
&= P\left(1 + \frac{1}{x}\right)^{xrt} \qquad \left(x = \frac{m}{r}\right) \\
&= P\left[\left(1 + \frac{1}{x}\right)^{x}\right]^{rt}
\end{aligned}$$

Continuous Compound Interest

$$A = P\left[\left(1 + \frac{1}{x}\right)^{x}\right]^{rt}$$

Continuous Compound Interest

$$A = P\left[\left(1 + \frac{1}{x}\right)^x\right]^{rt}$$

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

Continuous Compound Interest

$$A = P\left[\left(1 + \frac{1}{x}\right)^x\right]^{rt}$$

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

$$A = Pe^{rt}.$$

Continuous Compound Interest

Definition (Continuous Compound Interest)

Principal P invested at an annual nominal rate r will have future value

$$A = Pe^{rt}$$

after time t (in years).

Continuous Compound Interest

Definition (Continuous Compound Interest)

Principal P invested at an annual nominal rate r will have future value

$$A = Pe^{rt}$$

after time t (in years).

Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.